Role of Infrastructure Finance in Solow growth model

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# Questions for Solow Growth Model

| **Infrastructure** | Why do developing countries lack adequate infrastructure?  
|                    | Why does infrastructure wither away over time in developing countries? |
| **Reforms**        | What kind of reforms turn a low-income country into a middle income country |
| **Fiscal Policy**  | Does fiscal policy matter for long-run growth? |
| **Capital Flows**  | *Lucas (1990):* Developing countries have a lower capital labour ratio. So, why doesn’t capital flow to developing countries? |
What is capital in growth models?

**Physical capital**  very varied and impossible to aggregate

**Financial capital**  easy to aggregate

**Question**  What is *capital* in a growth model?

How many *types of capital* does a growth model require to capture the essence of reality?

**Answer**  …depends on how many *distinct channels* there are to transform output into capital
Output gets transformed into private capital through the saving channel
Output also gets transformed into *infrastructure* through the *fiscal* channel.
**Variables**

\[ Y = F(\bar{K}, K, AL) \]  
production function

- \( A \) labour augmenting technology
- \( K \) private capital
- \( \delta \) private capital’s depreciation rate
- \( \bar{K} \) public capital (infrastructure)
- \( \bar{\delta} \) public capital (infrastructure)’s depreciation rate
- \( s \) private saving rate
- \( \tau \) tax rate
- \( \varsigma \) public goods investment rate
- \( 1 - \varsigma \) leakage due to inefficiency and corruption
Output decomposed

\[ Y \]
output

\[ \varsigma \tau \cdot Y \]
output invested in public goods public capital (infrastructure)

\[ s(1 - \varsigma \tau) \cdot Y \]
output channeled to private savings and invested in private capital

\[ (1 - s)(1 - \varsigma \tau) \cdot Y \]
output consumed
Production function

\[ Y = \overline{K}^\beta K^\alpha (AL)^{1-(\alpha+\beta)} \]

Cobb-Douglas production function

\( \beta \) elasticity of output with respect to public capital

\( \alpha \) elasticity of output with respect to private capital

\[ y = \overline{k}^\beta k^\alpha \]

Cobb-Douglas production function (per-effective worker)

\( \overline{k} = \frac{\overline{K}}{AL} \) private capital per-effective worker

\( k = \frac{K}{AL} \) public capital per-effective worker
$k_{t+1} = s(1 - \varsigma \tau) \cdot \bar{k}^\beta k^\alpha + (1 - \delta - n - g)k_t$

$\bar{k}_{t+1} = \varsigma \tau \cdot \bar{k}^\beta k^\alpha + (1 - \bar{\delta} - n - g)\bar{k}_t$

Setting $\bar{k}_{t+1} = \bar{k}_t$ and $k_{t+1} = k_t$ gives us

$$\bar{k}(k) = \left[ \frac{\varsigma \tau}{\bar{\delta} + n + g} \right]^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}}$$

$$k(\bar{k}) = \left[ \frac{s(1 - \varsigma \tau)}{\delta + n + g} \right]^{\frac{1}{1-\alpha}} \bar{k}^{\frac{\beta}{1-\alpha}}$$
Proposition

For $\bar{k} \in (0, \infty)$ and $k \in (0, \infty)$, the economy has a unique steady $(\bar{k}^*, k^*)$ state where

$$\bar{k}^* = \left( \frac{s(1 - \varsigma \tau)}{\delta + n + g} \right)^{\frac{\alpha}{1-(\alpha+\beta)}} \left( \frac{\varsigma \tau}{\bar{\delta} + n + g} \right)^{\frac{1-\alpha}{1-(\alpha+\beta)}} \tag{1}$$

$$k^* = \left( \frac{s(1 - \varsigma \tau)}{\bar{\delta} + n + g} \right)^{\frac{1-\beta}{1-(\alpha+\beta)}} \left( \frac{\varsigma \tau}{\delta + n + g} \right)^{\frac{\beta}{1-(\alpha+\beta)}} \tag{2}$$

The steady state output and consumption per-effective worker is given by

$$y^* = \left( \frac{s(1 - \varsigma \tau)}{\delta + n + g} \right)^{\frac{\alpha}{1-(\alpha+\beta)}} \left( \frac{\varsigma \tau}{\bar{\delta} + n + g} \right)^{\frac{1-\alpha}{1-(\alpha+\beta)}} \tag{2}$$

$$c^* = (1 - s) \left( \frac{s^\alpha (1 - \varsigma \tau)^{1-\beta} (\varsigma \tau)^\beta}{(\delta + n + g)^\alpha (\bar{\delta} + n + g)^\beta} \right)^{\frac{1}{1-(\alpha+\beta)}} \tag{3}$$
Impact of Increase in $\zeta \tau$

$\overline{k}(k)$ shift right as public good investment increases

$k(\overline{k})$ shift down as disposable income decreases.
$k(\bar{k})$ shifts up.
SIMULATION OF OUTPUT, PUBLIC & PRIVATE CAPITAL

\[ \alpha = 0.10 \quad \beta = 0.30 \]

\[ k_t \]
\[ \bar{k}_t \]
\[ y_t \]

\( t = 30 \)
Simulation of output, public & private capital

\[ \alpha = 0.30 \quad \beta = 0.10 \]

- \( k_t \)
- \( \bar{k}_t \)
- \( y_t \)

\( t = 30 \)
Effect of Shock Increase Private Capital

\[ \alpha = 0.30 \quad \beta = 0.30 \quad s = 0.70 \quad t = 0.40 \]
The effect of shock increase in public

\[ \alpha = 0.30 \quad \beta = 0.30 \quad s = 0.70 \quad t = 0.40 \]
Main message

Saving channel and fiscal channel are inextricably linked
Per-capita income is determined by interaction of $s$ and $\zeta \tau$

Sustained per-capita income growth requires

increasing the fiscal capacity of the state
increasing the flow of tax collected into productive public goods
creating opportunities for private investment through infrastructure investment
increasing efficacy of the saving channel through financial reforms to take advantage of infrastructure investment