Reworking Chowdhury (2005)

Kumar Aniket
University of Cambridge
June 2014

We briefly describe the Chowdhury (2005) environment and compare the solution in Chowdhury (2005) with what I think should be the solution. Even though Chowdhury (2005) sets up a project-choice moral hazard model in the paper, he does not take the strategic interaction between the monitor and the borrower into account while solving the model. Chowdhury (2005) simply solves a model of costly state verification, i.e., the monitor invests in capacity that increases the probability of her finding finding the output. Further, when analysing group lending, the paper does not even model how lenders get the output when the output is discovered by the borrower's peer. Given that the investment in monitoring capacity does not have any impact on the borrower's project choice, my claim would be that there is no moral hazard component in the model. Chowdhury (2005) is a single task model of costly state verification.

1. Individual Lending

1.1. **Environment.** A borrower borrows 1 unit of capital from the lender at interest rate r to invest in a project. r, the interest rate the lender can charge is exogenously set by the government.[1]

The borrower has a choice about the kind of project she would like to pursue. The borrower can choose Project S, which produces output H which is verifiable through monitoring and Project R, which produces non-verifiable output b, which is not visible to anyone and cannot be discovered through monitoring.[2]

Project Choice	Output		Payoffs	
	verifiable	non-verifiable	borrower's	lender's
S	H		$H-r$	$r-1$
R		b	b	$0-1$

Table 1. Payoffs in Individual Lending

[1]Footnote 10 on page 419 in Chowdhury (2005).

[2]For the projects, it is easier to use the mnemonics of Project S and R used by Cason et al. (2012). Chowdhury (2005) uses P_k^1 and P_k^2, that is, Project 1 and Project 2, which is more confusing.

Assumption 1. $1 < r < H$.

Assumption 2. $H > b$.

Assumption 3. $H - r < b$.

The lender does not know if the borrower has chosen project S or R. If the lender invests monitoring cost $c(m) = \lambda \frac{m^2}{2}$ (where $\lambda > 1$), he discovers the output with probability $\min[m, 1]$ if the borrower chooses project S. If the borrower chooses project R, then the lender does not discover the output.

1.2. **Chowdhury (2005) Verbatim, Page 420-21.** The section sets out how Chowdhury (2005) solves the problem. The account is verbatim and taken from page 420-421.

`Start of the verbatim account.`

2.1. Individual Lending.

Stage 1: The bank decides whether to lend 1 dollar to an individual borrower. If the loan is made then the game goes to the next stage.
Stage 2: The bank decides on its level of monitoring m.
Stage 3: The borrower then invests the 1 dollar loaned earlier into one of the two projects.

We solve for the subgame perfect Nash equilibrium of this game.

Stage 3: The first project (S) is chosen if the bank is informed regarding the identity of the projects. In that case the bank gets back r, and the borrower obtains $H - r$. Otherwise the borrower chooses the second project (R). In that case the borrower gets b, but the bank does not obtain any repayment.
Stage 2: Consider the case where the bank has already lent 1 dollar to the borrower. Now the bank decides on the optimal level of monitoring. Note that the objective function of the bank

$$mr - \frac{\lambda m^2}{2} - 1$$

Clearly, the optimal level of bank monitoring $\hat{m} = \left(\frac{r}{\lambda}\right)$, the expected return of the bank is $\left(\frac{r^2}{2\lambda} - 1\right)$ and that of the borrower is $\left(\frac{r}{\lambda}\right)(H - r) + \left[1 - \left(\frac{r}{\lambda}\right)\right] b$.
Stage 1: Given that $H - r$, the expected profit of the borrower is strictly positive. Depending on the monitoring cost parameter, λ, the expected profit of the bank may, or may not be positive.

Summarizing the above discussion we can write down our first proposition.

Proposition 1. *Individual lending is feasible if and only if* $2\lambda < r^2$.[3]

Thus individual lending is feasible provided monitoring costs are not too large. Otherwise the bank does not have a sufficient incentive to monitor.

`End of the verbatim account.`

Chowdhury (2005) says that in Stage 3, "*The first project (S) is chosen if the bank is informed regarding the identity of the projects. . . . Otherwise the borrower chooses the second project (R).*" The fundamental flaw in this logic is that how can the bank know the identity of the project before the borrower has made her choice. The solution $\arg\max \left[mr - \frac{\lambda m^2}{2} - 1\right] = \frac{r}{\lambda}$ in Chowdhury (2005) in stead just assumes that the borrower chooses Project (S) by default. Put another way, the paper does not take the strategic interaction between the lender and borrower into account.

The solution presumes that the borrower will choose Project S in Stage 3 (and produce H verifiable output) with certainty, irrespective of the the lender's choice of m in the previous period. All the lender simply has to do is invest in monitoring capacity to maximise its probability of obtaining r from the verifiable output H.

Chowdhury (2005) and Cason et al. (2012) solve this model like a model of revenue maximisation where the lender invests in monitoring to balance the benefits from increasing the probability of extracting repayment on one hand and the cost of acquiring the monitoring capacity on the other hand. The choice of m does not take into account that it would effect the borrower's project choice in Stage 3. Thus, Chowdhury (2005) and Cason et al. (2012) completely ignore the moral hazard component of the model.

1.3. **My solution for Individual Lending.** This note tries to take the moral hazard component into account in my solution to the problem and the solution in this note thus differs from the Chowdhury (2005) and Cason et al. (2012) solution to the problem.

Solving the problem here backwards in the usual way gives us the following.

Stage 3: *Borrower chooses between Projects S and R taking lender's choice of m in Stage 2 as given.* Borrower's payoffs Π_B are function project choice $\{S, R\}$ and monitoring intensity chosen in Stage 2.

---

[3]In Chowdhury (2005), the opportunity cost of capital is 1 in individual lending and $\tilde{r}$ in group lending. Cason et al. (2012) corrects this. The opportunity cost of capital is $\bar{r}$ all through in their paper. That is why in Cason et al. (2012), the corresponding condition in Proposition 1 is $2\lambda < \frac{r^2}{\bar{r}+1}$.

§

$$\begin{aligned}\Pi_B(S,m) &= m(H-r)+(1-m)H \\ &= H - mr \\ \Pi_B(R,m) &= b\end{aligned}$$

Borrower chooses Project S over Project R if the following condition holds.[4]

$$\begin{aligned} H - rm &\geqslant b \\ \frac{H-b}{r} &\geqslant m \end{aligned}$$

Assumption 2 and 3 ensure that $\frac{H-b}{r} \in (0,1)$. The borrower's project choice is as follows.

$$\begin{cases} \text{Project } S, & \text{if} \quad m \in \left(0, \frac{H-b}{r}\right] \\ \text{Project } R, & \text{if} \quad m \in \left(\frac{H-b}{r}, 1\right] \end{cases} \tag{1}$$

This just means that the lender can spook into choosing Project R borrower by over-monitoring. The lender can ensure the choice of Project S by monitoring the borrower under the threshold $\frac{H-b}{r}$.

Stage 2: *The lender chooses m taking borrower's subsequent project choice in stage 3 into account.* The lender's payoff Π_L varies with m in the following way.

$$\Pi_L(m) = \begin{cases} mr - \lambda\frac{m^2}{2} - 1 & \text{if} \quad m \in \left(0, \frac{H-b}{r}\right] \text{ and borrower choose } S \\ -\lambda\frac{m^2}{2} - 1 & \text{if} \quad m \in \left(\frac{H-b}{r}, 1\right] \text{ and borrower choose } R \end{cases}$$

Project S is chosen if the monitoring is below the threshold $m = \frac{H-b}{r}$. Beyond the threshold $m = \frac{H-b}{r}$, Project R is chosen and the lender's monitoring is entirely wasted. Thus, there are two possible scenario. Either the lender choose an interior solution or a corner solution.

- The *interior solution* is $m = \arg\max\left[mr - \lambda\frac{m^2}{2} - 1\right] = \frac{r}{\lambda}$.
- The *corner solution* is $m = \frac{H-b}{r}$.

The lender chooses the interior solution if $\frac{r}{\lambda} < \frac{H-b}{r}$ and corner solution if $\frac{r}{\lambda} \geqslant \frac{H-b}{r}$.

It is useful to note here that by ignoring the strategic interaction, Chowdhury (2005) and Cason et al. (2012) ignore the corner solution.

[4]Assumption 3 implies that $\frac{H-b}{r} < 1$, which implies that this condition can be written as $1 > \frac{H-b}{r} \geqslant m$.

The lender's choice of optimal m can thus be summarised as follows.

$$m = \begin{cases} \frac{r}{\lambda} & \text{if} \quad \lambda > \frac{r^2}{H-b} \\ \frac{H-b}{r} & \text{if} \quad \lambda \leqslant \frac{r^2}{H-b} \end{cases}$$

It is useful to note that it follows from Assumption 1 and 3 that $\frac{r^2}{H-b} > 1$. Since, we assumed that $\lambda > 1$, both the interior and corner solutions are possible.

Lender's profit function Π_L is given below where it is easy to show that Π_L is continuously differentiable in λ.

$$\Pi_L\left(m\right) = \begin{cases} \frac{r^2}{2\lambda} - 1 & \text{if} \quad \lambda > \frac{r^2}{H-b} \\ \left(H-b\right)\left[1 - \frac{1}{2}\frac{\lambda(H-b)}{r^2}\right] - 1 & \text{if} \quad \lambda \leqslant \frac{r^2}{H-b} \end{cases} \tag{2}$$

State 1: Bank chooses whether to lend or not. The bank will only lend only if it at least breaks even, i.e., $\Pi_L \geqslant 0$. From (2) if follows that $\Pi_L \geqslant 0$ if the following conditions hold.

- If $\lambda > \frac{r^2}{H-b}$, then $\Pi_L \geqslant 0$ holds only when $\frac{r^2}{2} \geqslant \lambda$.
- If $\lambda \leqslant \frac{r^2}{H-b}$, then $\Pi_L \geqslant 0$ holds only when $\left(\frac{H-b-1}{H-b}\right)\frac{2r^2}{H-b} \geqslant \lambda$.

The two conditions above can be written as

$$\frac{r^2}{2} \geqslant \lambda > \frac{r^2}{H-b} \tag{3}$$

$$\min\left[2\left(\frac{H-b-1}{H-b}\right)\frac{r^2}{H-b}, \frac{r^2}{H-b}\right] \geqslant \lambda \tag{4}$$

Thus, the lender would choose to lend under the following condition.[5]

- If $2 \geqslant H-b$ and $2\left(\frac{H-b-1}{H-b}\right)\frac{r^2}{H-b} \geqslant \lambda$.
- If $H-b > 2$ and $\frac{r^2}{2} \geqslant \lambda$

You only get the result in Proposition 1 of Chowdhury (2005) if $H-b > 2$. From Assumption 1 and 3 we know that $H-b < r$ and that $1 < r < H$. Thus, result in Proposition 1 of Chowdhury (2005) is only possible if both $H-b$ and r are greater than 2.

2. Discussion

Chowdhury (2005), if solved in my way, is clearly reminiscent of the Besley and Coate (1995). The problem Besley and Coate (1995) tackles is one of enforcing contracts. In Besley and Coate (1995), the realisation of the output is stochastic and the lender's penalty

[5] $H-b > 2$ implies that (4) becomes $\frac{r^2}{H-b} \geqslant \lambda$ which implies that from (3) and (4) we get $\frac{r^2}{2} \geqslant \lambda$. Conversely, if $2 > H-b$, $\frac{r^2}{H-b} > \frac{r^2}{2}$, which implies that the range for λ in (3) does not exist. In this case, (4) becomes $2\left(\frac{H-b-1}{H-b}\right)\frac{r^2}{H-b} \geqslant \lambda$. That is only the corner solution is possible if $2 > H-b$.

function[6] increases with output. For a given interest rate, the borrower only repays if the penalty is greater than interest rate, which happens when the output realisations are higher than a particular threshold. If output realisation are below this threshold, the borrower just prefers to default and bear the cost of the penalty.

In the solution in Chowdhury (2005), there is monitoring threshold $\left(\frac{H-b}{r}\right)$ beyond which the borrower switches from Project S or R. The lender wants to ensure that he does not spook the borrower by over-monitoring (1). Thus, the lender would restrict himself to monitoring below the threshold $\frac{H-b}{r} \in (0,1)$. My version of Chowdhury (2005) can be read as a model where the lender invests ex-ante in revenue extraction capacity m. The investment in revenue extraction capacity is costly and the lender uses the revenue extraction capacity to influence the borrower's subsequent actions.

In principle, the investment in revenue extraction works quite like the way penalty function works in Besley and Coate (1995). The difference between the two models is that trigger in Besley and Coate (1995) is an exogenous stochastic process, where as in my version of Chowdhury (2005), the trigger is endogenous and determined by the lender.

The way Chowdhury (2005) solves the model, the borrower does not take the lender's choice of m into account. The lender just chooses m to maximises his profit function presuming that the borrower would choose Project S. Thus, it is a solution to the simple problem of revenue maximisation with no strategic interaction and no moral hazard.

2.1. **Group Lending.** In group lending, it is the borrowers that choose the m. Both Chowdhury (2005) and Cason et al. (2012) continue to ignore impact of m on project choice determined at the later stage of the game.

Further, in Chowdhury (2005) and Cason et al. (2012), the lender does not invest in any monitoring capacity. Thus, the output is not visible to the lender even if the borrowers choose Project S. The lender using group lending and the intra-group social sanction to leverage their own enforcement capacity is a theme in the papers in this area, i.e., papers like Besley and Coate (1995) and Rai and Sjöström (2004). Chowdhury (2005) and Cason et al. (2012) do not explain how the lender would know if the borrowers have discovered the output. Further, how would he extract the output from the borrowers, even if the borrower invest in monitoring capacity and are able to find the revenue stream from the Project S.

As we show below, taking the strategic interaction into account turns out to be extremely complicated with numerous sub-cases. Instead, Chowdhury (2005) and Cason et al. (2012) just stick to the interior solutions for m.

2.2. **Simultaneous Group Lending.** Verbatim from (Chowdhury, 2005, Page 422)

---

[6] or the social penalty function

`Start of the verbatim account.`

We then solve for the subgame perfect Nash equilibrium of this game.

Stage 3: If both the borrowers are successful in monitoring then they mutually ensure that they both invest in the first project. Then the bank gets back $2r$, and both the borrowers get $H-r$. If, however, both the borrowers fail in their monitoring efforts then they both invest in the second project. In that case both the borrowers obtain b, while the bank obtains nothing. ...

Stage 2: Next we solve for the Nash equilibrium of the game where the borrowers simultaneously decide on their level of monitoring. Clearly, the net payoff of the ith borrower is

$$m_i m_j (H-r) + m_i(1-m_j)b + (1-m_i)(1-m_j)b - \frac{m_i^2}{2}$$

Hence the reaction function of the ith borrower is given by

$$m_i = m_j(H-r)$$

`End of the verbatim account.`

My comment. This is exactly the way Cason et al. (2012) approaches the problem on page 195 of their article. The presumption here is than any positive monitoring in Stage 2 stage will lead the borrower to choose Project S over Project R in a Stage 3. As we discussed above, the problem in these class of models is one of spooking the borrowers by over-monitoring them.

2.3. **My Solution.** In stage 3, the borrowers find themselves in subgame $\xi(m_i, m_j)$ where they have already chosen their monitoring levels m_i and m_j in stage 2. Both borrowers choose their projects simultaneously in subgame $\xi(m_i, m_j)$. We abuse the notation to let S_i represents the situation when i chooses project S. Similarly, R_i represents i choosing project R.

Borrower i's payoff from choosing project S when j has chosen project S is given below.

$$\begin{aligned}\Pi_i\left(S_i, m_i, \bar{S}_j, \bar{m}_j\right) &= m_i\bar{m}_j(H-r) + m_i(1-\bar{m}_j)H + (1-m_i)\bar{m}_j 0 + (1-m_i)(1-\bar{m}_j)H - \frac{m_i^2}{2} \\ &= m_i m_j(H-r) + (1-m_j)H - \frac{m_i^2}{2}\end{aligned}$$

§

Project Choice	Output		Payoffs	
	verifiable	non-verifiable	each borrower's	lender's
SS	$2H$		$H-r, H-r$	$2r$
SR	H	b	$0, b$	H
RS	H	b	$b, 0$	H
RR		$2b$	b, b	0

TABLE 2. Group Lending: First and Second Agent's Project choices and Payoffs

Similarly, we find borrower i's payoff in subgame $\xi(m_i, m_j)$ when i and j choose project pairs (SR), (RS) and (RR) respectively.

$$\begin{aligned}\Pi_i\left(S_i, m_i, \bar{R}_j, \bar{m}_j\right) &= m_i\bar{m}_j 0 + m_i(1-\bar{m}_j)H + (1-m_i)\bar{m}_j 0 + (1-m_i)(1-\bar{m}_j)H - \frac{m_i^2}{2} \\ &= (1-m_j)H - \frac{m_i^2}{2}\end{aligned}$$

$$\begin{aligned}\Pi_i\left(R_i, m_i, \bar{S}_j, \bar{m}_j\right) &= m_i\bar{m}_j b + m_i(1-\bar{m}_j)b + (1-m_i)\bar{m}_j b + (1-m_i)(1-\bar{m}_j)b - \frac{m_i^2}{2} \\ &= b - \frac{m_i^2}{2}\end{aligned}$$

$$\begin{aligned}\Pi_i\left(R_i, m_i, \bar{R}_j, \bar{m}_j\right) &= m_i\bar{m}_j b + m_i(1-\bar{m}_j)b + (1-m_i)\bar{m}_j b + (1-m_i)(1-\bar{m}_j)b - \frac{m_i^2}{2} \\ &= b - \frac{m_i^2}{2}\end{aligned}$$

The payoff matrix of the subgame $\xi(m_i, m_j)$ is represented below.

		Player j: S_j	Player j: R_j
Player i	S_i	$m_im_j(H-r) + (1-m_j)H - \frac{m_i^2}{2}$ $m_im_j(H-r) + (1-m_i)H - \frac{m_j^2}{2}$	$(1-m_j)H - \frac{m_i^2}{2}$ $(1-m_j)H - \frac{m_i^2}{2}$
	R_i	$b - \frac{m_i^2}{2}$ $b - \frac{m_j^2}{2}$	$b - \frac{m_i^2}{2}$ $b - \frac{m_j^2}{2}$

It follows from above that i will choose S over R if

$$m_j(m_i - \alpha) \geqslant \beta \tag{5}$$

and that j will choose S over R if

$$m_i(m_j - \alpha) \geqslant \beta \tag{6}$$

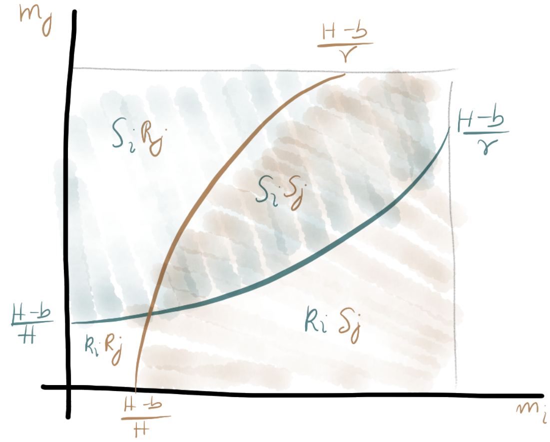

FIGURE 1. Monitoring Intensities and Project Choices in Simultaneous Lending

where $\alpha = \frac{H}{H-r} \quad \beta = \frac{b-H}{H-r}$. (5) and (6) are both rectangular hyperbole functions and satisfied only in a lens shaped area of the Figure 1.

Borrower i's payoff in the Stage 2 of the game is given below.

$$\Pi_i = \begin{cases} = m_i m_j (H-r) + (1-m_j)H - \frac{m_i^2}{2} & \text{if} \quad m_i(m_j - \alpha) \geqslant \beta \text{ and } m_j(m_i - \alpha) \geqslant \beta \\ = b - \frac{m_i^2}{2} & \text{if} \quad m_i(m_j - \alpha) < \beta \text{ and } m_j(m_i - \alpha) < \beta \\ = (1-m_j)H - \frac{m_i^2}{2} & \text{if} \quad m_j(m_i - \alpha) \geqslant \beta \text{ and } m_i(m_j - \alpha) < \beta \\ = b - \frac{m_i^2}{2} & \text{if} \quad m_j(m_i - \alpha) < \beta \text{ and } m_i(m_j - \alpha) \geqslant \beta \end{cases}$$

Looking at Stage 1, the lender's payoff is given by:

$$\Pi_b = m_i m_j (2r) + m_i(1-m_j)H + m_j(1-m_i)H + (1-m_i)(1-m_j)0$$

The lender would lend to the group in stage 1 one if it breaks-even.[7]

$$\Pi_b = m_i m_j (2r) + m_i(1-m_j)H + m_j(1-m_i)H - 2 \geqslant 0$$

As we can see there are a number of subgame perfect equilibria of this game. Proposition 2 on page 423 in Chowdhury (2005) says that "*Group Lending is not feasible*" does not seem to be accurate. Similarly, Proposition 2 on Page 196 in Cason et al. (2012) that states that "*If $H - r > 1$ and agents coordinate on the payoff-dominant Nash equilibrium, then under a simultaneous group lending scheme lenders choose to make loans, borrowers choose a high level of monitoring and repayment rates are high leading to an efficient (monitoring/lending)*

[7]We are sticking to the assumption that opportunity cost of 1 unit of capital is 1.

equilibrium. However, an inefficient zero-monitoring equilibrium with no lending also exists." seem inaccurate.

2.4. **Sequential Group Lending.** Verbatim from (Chowdhury, 2005, Page 426).

`Start of the verbatim account.`

Case A: As usual we solve the game through backwards induction. Let m denote the equilibrium level of monitoring for both the borrowers in this case. Straightforward calculations show that[8]

$$\tilde{m} = \min\left\{1, \frac{\alpha(H-r)(1+\tilde{r})}{2\tilde{r}}\right\}$$

In case of an interior solution $\bar{m}$ is increasing in H and decreasing in both r and $\tilde{r}$. Moreover, the equilibrium payoff of both the borrowers is

$$\frac{b}{2}(1-\bar{m}) + \bar{m}\left[\frac{\alpha(H-r)(1+\tilde{r})}{4\tilde{r}} - \frac{\bar{m}}{2}\right] \geqslant 0$$

and the bank is

$$\bar{m}\left(r + \frac{r+\tilde{r}-1}{\tilde{r}} - 1\right) - 1$$

Note that in this case the payoff of the borrowers is increasing in b, whereas the payoff of the bank is independent of b. Clearly, in this case group-lending is feasible if and only if $\bar{m}(r + (\frac{r+\tilde{r}-1}{r}) - 1) - 1 > 0$.

Thus we find that the monitoring level is strictly positive and moreover, for some parameter values, group-lending is feasible.

The equilibrium outcomes in cases B and C are qualitatively similar.

Interestingly we find that there is a positive level of monitoring irrespective of the value of α, i.e. irrespective of the nature of side-contracting between the two borrowers. The intuition is as follows. Consider the problem facing, say, borrower 1. Even if borrower 2 does not monitor, borrower 1 has a positive incentive to monitor. Suppose that borrower 2 receives the loan in period 1. If borrower 1 does not monitor, then borrower 2 would invest in P_2^2 and borrower 1 would have a payoff of zero. By monitoring, however, she may force borrower 2 to invest in P_2^1 when the group gets an additional loan in the second period which comes to borrower 1. Moreover, given that borrower 1 is going to monitor, borrower 2 now has a greater incentive to monitor herself, and so on. ...

`End of the verbatim account.`

[8] The detailed derivation of this case, along with that of cases B and C has been relegated to Appendix A.

§

My comment. Again, Chowdhury (2005) paper continues to presume that any monitoring at all will ensure that the borrower in the subsequent stage will choose Project S over R. They don't take into account that over-monitoring will spook the borrower to choose Project R.

References

Besley, T. J. and Coate, S. (1995). Group lending, repayment incentives and social collateral. *Journal of Development Economics*, 46(1):1–18.

Cason, T. N., Gangadharan, L., and Maitra, P. (2012). Moral hazard and peer monitoring in a laboratory microfinance experiment. *Journal of Economic Behavior & Organization*, 82(1):192–209.

Chowdhury, P. R. (2005). Group-lending: Sequential financing, lender monitoring and joint liability. *Journal of Development Economics*, 77:415–439.

Rai, A. S. and Sjöström, T. (2004). Is grameen lending efficient? repayment incentives and insurance in village economies. *Review of Economic Studies*, 71:217–234.

Faculty of Economics, University of Cambridge, Sidgwick Avenue, Cambridge CB3 9DD.
E-mail address: `ka323@cam.ac.uk`